SPECIMEN ACADEMICUM
DE
INVENIENDA SECTIONE CONICA CIRCA FOCUM DATUM, PER DATA TRIA PUNCTA TRANSEUNTE.

QUOD
CONS. AMPL. FAC. PHILOS. ABOËNS.
PRÆSIDE
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PUBLICÆ CENSURÆ SUBMITTIT
JACOBUS WEGELIUS,
OSTROBOTNIENSIS,
IN *AUDITORIO MINORI*
DIE XVII JUNII MDCCLXXXVI.
H. A. M. C.

ABOÆ, TYPIS FRENCKELLIANIS.

§. I.

Inter varias Methodos, quibus orbitam Planetæ cujusvis vel Cometæ detegere solent Astronomi, locum haud ultimum meretur illa, qua ex diversis observationibus primo investigantur tria ejus loca heliocentrica seu vera. His videlicet inventis, quum omnes planetæ primarii atque cometæ circa solem, ut & satellites circa suos primarios, in Sectionibus Conicis moveantur, eo reducta est quæstio, ut determinetur Sectio Conica circa datum focum, per tria data puncta transiens. Hoc problema ab EDMUNDO HALLEY A. 1676 primo propositum atque geometrice solutum est. Constructio vero ab Illo tradita quum difficilior sit, quippe intersectionem duarum hyperbolarum supponens, aliam faciliorem recta & circulo perficiendam docuit PH. DE LA HIRE, quam videre licet in Ejus *Sect. Conic. Lib. VIII. Prop. 25.* Omnium autem simplicissima atque maxime concinna nobis videtur constructio geometrica hujus problematis, quam proprietatibus directricium fundatam dedit magnus NEWTONUS in *Philos. Nat. Princ. Math. Lib. I. prop. 21. Schol.* In quavis scilicet Sectione Conica *AML* (Fig. 1.), cujus sit vertex *A*, Focus *S*, si in axe

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transversi producto capiatur AS ad AB in ratione eccentricitatis ad semiaxem, seu AB ad SB ut AS ad semiparametrum principalem, & ex B eidem axi perpendicularis ducatur Bl, erit hæc Bl linea, quæ directrix, aliis linea sublimitatis dicitur, & cujus hæc est proprietas, ut ex quibuscunque Sectionis Conicæ punctis L, M, N, ductis rectis LS, MS, & NS ad Focum, nec non Ll, Mm, & Nn perpendicularibus ad directricem, sit semper $Ll : LS :: Mm : MS :: Nn : NS :: AB : AS$. Rectis igitur per L & M, nec non per M & N productis donec directrici occurrant in P & Q, quum sit $\Delta\, MmP \backsim \Delta\, LlP$, atque $\Delta\, NnQ \backsim \Delta\, MmQ$, sequitur fore $PM : PL :: Mm : Ll :: MS : LS$, & $QN : QM :: Nn : Mm :: NS : MS$, adeoque (divisim) $PM : ML :: MS : LS - MS$, nec non $QN : NM :: NS : MS - NS$. Hinc facile elicitur problematis nostri constructio NEWTONIANA. Datis videlicet punctis L, M, N, atque Umbilico S, dantur ope harum proportionum P & Q, adeoque Directrix Ql, cui si ex Foco agatur normalis SB, dabitur positio axeos transversi. Hujus denique inveniuntur vertices, sumendo $SA : BA$ in ratione data $SN : Nn$; quo facto determinata est Sectio Conica, quæ quidem, prout fuerit AB vel major vel æqualis vel minor quam AS, erit aut Ellipsis aut Parabola aut Hyperbola.

§. II.

Constructiones Geometricæ, utcunque elegantes atque concinnæ, in re tamen Astronomica exigui sunt usus, nisi regulæ ex iis simul derivari queant, ad calculum dirigendum idoneæ. Quando autem invenienda est sectio Conica circa datum Focum S (Fig. 1.) per puncta data L, M, N transiens, communiter determinari solent hæc tria puncta per radios vectores ad solem seu Focum ductos LS, MS, NS, atque angulos ab his radiis comprehensos LSM, MSN. In præsenti igitur negotio præcipue ostendendum est, quomodo ex datis tribus radiis vectoribus cum angulis interceptis, computatione inveniantur Sectionis Conicæ situs & dimensiones. Methodum, qua ad calculum revocari possit constructio illa NEWTONIANA (§. I.) jam (*Introd. ad ver. Astron. Lect. XXVI p. m.* 465) docuit JOH. KEILL. Analysi scilicet Trigonometrica ex datis duobus lateribus cum angulo intercepto in utroque $\Delta\Delta$ LSM, MSN, inveniuntur LM, & ang. LMS, nec non MN & ang. NMS. Datis vero angulis SML, SMN, datur ang. PMQ. Inventis præterea LM & MN, per regulam proportionum (§. I.) innotescunt PM & MQ; unde porro ex cognitis in Δ PMQ duobus lateribus & angulo ab his comprehenso investigatur alteruter reliquorum angulorum PQM ejusque Complementum mMQ. Huic si addatur notus angulus NMS, dabitur mMS, adeoque etiam

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hujus supplementum, scil. ang. MSB, quo positio axeos determinatur. Ex datis autem angulis & latere MQ in $\triangle\, QMm$ rectangulo ad m, invenitur Mm, & hinc $BS = Mm - MS\ Cos\ SMm$ (assumto Sinu toto $= 1$). Quumque (§. I.) sit $Mm : MS :: AB : AS$ seu ut semiaxis principalis ad eccentricitatem, erit (comp.) $Mm + MS : MS :: BS : AS$ & (div.) $Mm - MS : MS :: AS$ ad eccentricitatem, unde vertex A & Centrum C sectionis Conicæ inveniuntur.

§. III.

Methodus hæc KEILLIANA (§. II.) quamvis primo intuitu admodum simplex videatur, computationem tamen exigit nimis tædiosam. Hanc calculi prolixitatem evitaturus Dn. NICOLLIC ad solvendum problema nostrum novam ingressus est viam (*Mem. de l'Acad. Roï. des Sciences de Paris An. 1746 p. 291-318*) Fundamenti nempe loco ponit hoc Lemma: Si fuerint puncta N, M, L (Fig. 2.) posita in perimetro Sectionis Conicæ, cujus Focus S, & centro S radio quocunque describatur circulus nmL, radios vectores SN, SM, SL, productos, si opus fuerit, secans in n, m, L, ducanturque rectæ LM, & LN, nec non his correspondentes chordæ circuli Lm & Ln, quæ in E & e secentur in ratione inversa radiorum contiguorum, ita scil. ut sit $LE : Em :: MS : LS$ & $Le : en :: NS : LS$, atque agantur ex E in LM, &

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ex e in Ln, perpendiculares EK, eK se mutuo decussantes in K: erit I:mo hoc punctum K positum in axe transverso ejusdem Sectionis & quidem ad partem oppositam a Foco S respectu verticis huic foco proximi; 2:do erit SK ad radium circuli SL, ut eccentricitas sectionis ad semiaxem transversum; & 3:tio ductæ ex K ad puncta circuli L, m, n, rectæ perpendiculariter insistent rectis sectionem Conicam in punctis L, M, N contingentibus respective. His principiis fuse expositis Dn. NICOLLIC duplicem superstruit problematis propositi solutionem, alteram trigonometricam, alteram Algebraicam. Datis videlicet magnitudine & positione tribus radiis vectoribus SL, SM, SN, quorum sit SL maximus, descripto per L centro S, circulo Lmn, & facta constructione modo descripta, nec non demissis ex S in Chordas Lm, Ln normalibus SD, Sd: quum manifestum sit fore ang. $MLm = \frac{1}{2}(LMS - SLM)$, erit (Elem. Trigon.) $Tg\ MLm : Cotg\ \frac{1}{2}\ LSM :: SL - SM : SL + SM$. Porro quum (Constr.) sit $SL : SM :: mE : EL$, erit $SL + SM : SL - SM :: DL : DE :: Tg\ DSL\ (= Tg\ \frac{1}{2}\ LSM) : Tg\ DSE$. Hinc innotescunt anguli MLm & DSE, horumque complementa LEK & DES, adeoque angulus SEK. Pari ratione est $SL + SN : SL - SN :: Cotg\ \frac{1}{2}\ LSN : Tg\ NLn :: Tg\ \frac{1}{2}\ LSN : Tg\ eSD$, unde obtinentur anguli NLn & eSD cum suis complementis LeK & SeD, adeoque ang. SeK. Præterea in $\Delta\ LES$ est $Sin\ DES : Sin\ ELS\ (= Cos\ \frac{1}{2}\ LSM) :: LS : SE$ & in $\Delta\ LeS$, $Sin\ DeS : Sin\ eLS$

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$(= Cos \frac{1}{2} LSN) :: LS : Se$, quamobrem data sunt ES & eS. Ulterius in $\Delta\Delta$ ESK, eSK est $ES : SK :: Sin\ EKS : Sin\ SEK$ & $SK : eS :: Sin\ SeK : Sin\ eKS$, quibus rationibus compositis efficitur $ES : eS :: Sin\ EKS . Sin\ SeK : Sin\ eKS . Sin\ SEK$ seu $ES . Sin\ SEK : eS . Sin\ SeK :: Sin\ EKS : Sin\ eKS$. Hinc posito $ES . Sin\ SEK : eS . Sin\ SeK :: Tg\ x : 1$ adeoque $Tg\ x : 1 :: Sin\ EKS : Sin\ eKS$, erit (comp. & div.) $1 : Tg\ (45^\circ - x) :: Tg \frac{1}{2} (EKS + eKS) : Tg \frac{1}{2} (EKS - eKS)$; unde ob cognitum ex præced. angulum EKe, dantur singuli anguli EKS, eKS & hinc anguli ESK, eSK, adeoque situs Axeos: datisque præterea (dem.) rationibus $SK : SE$ & $SE : SL$, datur ratio eccentricitatis ad semiaxem $= SK : SL$. Huic Methodo Trigonometricæ aliam subjungit Algebraicam, cujus hæc est summa. Posito $SK : SL :: e : 1$, quum sit $SL : SN :: 1 - e\ Cos\ NSK : 1 - e\ Cos\ LSK$ atque $SL : SM :: 1 - e\ Cos\ MSK : 1 - e\ Cos\ LSK$: sequitur fore $e : 1 :: SL - SN : SL . Cos\ LSK - SN . Cos\ NSK :: SL - SM : SL\ Cos\ LSK - SM . Cos\ MSK$, unde facta debita reductione invenitur: $Tg\ (MSK + \frac{1}{2} MSN)$

$$= \frac{\left(\frac{LS}{NS} - 1\right) - \left(\frac{LS}{MS} - 1\right)}{\left(\frac{LS}{NS} - 1\right) Cotg \frac{1}{2} LSN + \left(\frac{LS}{MS} - 1\right) Cotg \frac{1}{2} LSM.}$$

Computatis vero secundum alterutram harum Methodorum ang. LSK & ratione $e : 1$, obtinetur tandem semiparameter orbitæ $= LS . (1 - e\ Cos\ LSK)$.

§. IV.

§. IV.

Quum analysi nimium prolixa ad eruendas regulas (§. III.) allatas usus fuerit Dn. NICOLLIC, aliam multo simpliciorem dedit Celeb. Prof. & Astron. Reg. Dn. PROSPERIN in *Nov. Act. Upsal. Vol. III. p. 257-261.* Posita videlicet ut supra ratione eccentricitatis ad semiaxem transversum $= e : 1$, quum sit Sectionis Conicae AML (Fig. 1.) circa Focum S per verticem A descriptae, semiparameter $= SN\,(1 + e\,Cos\,NSA) = SM\,(1 + e\,Cos\,MSA) = SL\,(1 + e\,Cos\,LSA)$, sequitur fore: $\frac{MS - NS}{NS\,Cos\,NSA - MS\,Cos\,MSA}$ $= e = \frac{LS - NS}{NS\,Cos\,NSA - LS\,Cos\,LSA}$: unde porro facili reductione obtinetur: $Tg\,NSA =$

$$\frac{\left(1 - \frac{NS}{LS}\right) Cos\,NSM - \left(1 - \frac{NS}{MS}\right) Cos\,LSN - \left(\frac{NS}{MS} - \frac{NS}{LS}\right)}{\left(1 - \frac{NS}{LS}\right) Sin\,NSM - \left(1 - \frac{NS}{MS}\right) Sin\,LSN}$$

Harum formularum ope inveniuntur angulus NSA, nec non e & orbitae parameter, ex quibus porro investigari potest distantia perihelii seu AS, quippe quae est ad semiparametrum $: : 1 : 1 + e$, nec non semiaxis major, qui est ad $AS : : 1 : 1 - e$, atque axis minor qui inter axem majorem hujusque parametrum media proportionalis est. Si in invento valore $Tg\,NSA$

feſequentes fiant ſubſtitutiones: $(LS - MS) NS = (LS - NS) MS - (MS - NS) LS$; $1 - Cof\, NSM = 2\, Sin\, \tfrac{1}{2} NSM^2$; $1 - Cof\, LSN = 2\, Sin\, \tfrac{1}{2} LSN^2$; $Sin\, NSM = 2\, Sin\, \tfrac{1}{2} NSM\, Cof\, \tfrac{1}{2} NSM$ & $Sin\, LSN = 2\, Sin\, \tfrac{1}{2} LSN\, Cof\, \tfrac{1}{2} LSN$; formula illa in hanc transformatur: $Tang\, NSA =$

$$\frac{\left(1 - \frac{NS}{LS}\right) Sin\, \tfrac{1}{2} NSM^2 - \left(1 - \frac{NS}{MS}\right) Sin\, \tfrac{1}{2} LSN^2}{\left(1 - \frac{NS}{LS}\right) Sin\tfrac{1}{2} NSM\, Cof\tfrac{1}{2} NSM - \left(1 - \frac{NS}{MS}\right) Sin\tfrac{1}{2} LSN\, Cof\tfrac{1}{2} LSN}$$

Cujus cum formula (§. III.) Dn. Nicollic conſenſum facile eſt detegere ope notiſſimi Theorematis Trigonometici: $Tg\, (\alpha + \beta) = \frac{Tg\, \alpha + Tg\, \beta}{1 - Tg\, \alpha\, Tg\, \beta}$.

§. V.

Expoſitis in §§. præced. diverſis methodis inveſtigandi Sectionem Conicam, circa datum umbilicum per tria data puncta transeuntem, ſuperfluum forte videbitur novam aliquam ad eundem ſcopum ducentem quærere viam. Nec ſolutionem tentare, novis quibusdam principiis vel antea non detectis ſectionum Conicarum proprietatibus ſuperſtructam, in animo nobis eſt. Satis habemus, ſi methodus a nobis jam adferenda in applicatione aliquo calculi compendio ſeſe commendet. Sint igitur (Fig. 1.) data puncta *L*, *M*, *N*, atque Focus *S*, & invenire oporteat ipſam ſectionem $LMNA$, cujus vertex dicatur A & Centrum C. Ponantur $SL = R$, $SM = r$, $SN = \varrho$; ang. $LSM = 2v$, ang. $LSN = 2u$; semiparameter principalis $= b$, semiaxis transversus $CA = a$, eccentricitas $CS = ae$ (seu $CA : CS :: 1 : e$) atque ang. $CSL = z$, qui quidem angulus (existente ANL orbita planetæ circa solem S) anomalia vera puncti L apud Astronomos nominari solet. His denominationibus assumtis, (facto ubique Sinu toto $= 1$) erit ob notissimam Sectionum Conicarum proprietatem $b = R\,(1 - e\,Cos\,z)$ seu $\frac{b}{eR} = \frac{1}{e} - Cos\,z$ (A). Pari ratione $\frac{b}{er} = \frac{1}{e} - Cos\,(z + 2v)$ (B) & $\frac{b}{e\varrho} = \frac{1}{e} - Cos\,(z + 2u)$ (C). Si jam ab æquatione A subtrahantur æqu. B & C, (ob $Cos\,\alpha - Cos\,\beta = 2\,Sin\,\frac{1}{2}\,(\beta - \alpha)\,Sin\,\frac{1}{2}\,(\beta + \alpha)$) obtinentur $\frac{b\,(R - r)}{eRr} = 2\,Sin\,v\,Sin\,(z + v)$ (D), & $\frac{b\,(R - \varrho)}{e\varrho R} = 2\,Sin\,u\,Sin\,(z + u)$ (E); unde sumta $Tg\,\lambda = \frac{(R - r)\,\varrho\,Sin\,u}{(R - \varrho)\,r\,Sin\,v}$ (I.), sequitur fore $1 : Tg\,\lambda :: Sin\,(z + u) : Sin\,(z + v)$, adeoque (comp. & divid.) $1 + Tg\,\lambda : 1 - Tg\,\lambda :: Sin\,(z + u) + Sin\,(z + v) : Sin\,(z + u) - Sin\,(z + v)$, hoc est: $Tg\,(45^\circ + \lambda) : 1 :: tg\,\overline{z + \frac{1}{2}\,(u + v)} : tg\,\frac{1}{2}\,(u - v)$ seu $tg\,\overline{z + \frac{1}{2}\,(u + v)} = Tg\,\frac{1}{2}\,(u - v)\,Tg\,(45^\circ + \lambda)$ (II.). Datis igitur positiositione & magnitudine tribus radiis vectoribus SL, SM, & SN, positio axeos determinari potest ope harum formularum:

$$\text{I. } Tg\,\lambda = \frac{(R - r)\,\varrho\,Sin\,u}{(R - \varrho)\,r\,Sin\,v}$$

$$\text{II. } Tg\,[z + \tfrac{1}{2}(u + v)] = Tg\,\tfrac{1}{2}(u - v)\,Tg\,(45^{\circ} + \lambda).$$

Scholion. Vel nobis non monentibus patet, si vel alterutrum vel utrumque punctorum M, N, ad alteram partem ipsius L a vertice cadat, angulos his respondentes v, u, negative sumendos esse, unde facilis est applicatio nostrarum formularum, si potius anomaliam veram alterutrius reliquorum punctorum primo investigare placeat.

§. VI.

Inventa sic positione axeos, dispiciendum ulterius est, qua ratione dimensiones hujus sectionis Conicæ investigari queant. Quum vero (§. V. A.) sit $\frac{1}{e} = \frac{b}{eR}$ $+ Cos\,z$, substituendo in hac æqu. pro $\frac{b}{eR}$ valorem ejus ex alterutra æqu. D vel E (§. V.) fiet $\frac{1}{e}$ $=$

$$\frac{2\,r\,Sin\,v\,Sin\,(z + v)}{R - r} + Cos\,z = \frac{2\,\varrho\,Sin\,u\,Sin\,(z + u)}{R - \varrho}$$

 $+ Cos$

$+ Cos\, z$ (F). Si igitur $\frac{2\,r\, Sin\, v\, Sin\,(z+v)}{R-r}$ vel huic æqualis $\frac{2\,\varrho\, Sin\, u\, Sin\,(z+u)}{R-\varrho}$ ponatur $=Cotg\, x\, Sin\, z$, hoc est:

$$\frac{(R-r)\, Sin\, z}{2\,r\, Sin\, v\, Sin\,(z+v)} = Tg\, x = \frac{(R-\varrho)\, Sin\, z}{2\,\varrho\, Sin\, u\, Sin\,(z+u)} \text{ (III.)}$$

æquatio F in hanc transformatur: $\frac{1}{e} = Cotg\, x\, Sin\, z + Cos\, z = \frac{Cos\, x\, Sin\, z + Sin\, x\, Cos\, z}{Sin\, x} = \frac{Sin\,(z+x)}{Sin\, x}$ vel:

$$e = \frac{Sin\, x}{(Sin\, z+x)} \text{ (IV.)}$$

Unde simul innotescit cujus generis sit ipsa sectio Conica; prout videlicet fuerit $e <=> 1$, illa erit aut Ellipsis, aut Parabola aut Hyperbola. Hunc valorem e iterum substituendo in æqu. D vel E, (§. V.) obtinetur b, cujus vero valor exterminando r vel ϱ ope æqu. III. simplicior fiet, scilicet:

$$b = \frac{R\, Sin\, z\, Cos\, x}{Sin\,(z+x)} \text{ (V.)}$$

Quumque in omni sectione Conica sit $a\,(1-e^2) = b$ adeoque per æqu. IV $a\,(\overline{Sin\, z+x}^2 - Sin\, x^2) = b\, Sin\, \overline{z+x}^2$, hoc est: $a\, Sin\, z\, Sin\,(z+2x) = R\, Sin\, z\, Cos\, x\, Sin\,(z+x)$; erit

$a =$

$$a = \frac{R.\, Cos\, x\, Sin\, (z + x)}{Sin\, (z + 2x)} \quad \text{(VI.)}$$

Si ſemiaxis ſectionis conjugatus dicatur c, quum ſit $c^2 = ab$, ſequitur ex form. V & VI fore:

$$c = R\, Cos\, x \sqrt{\frac{Sin\, z}{Sin\, (z + 2x)}} \quad \text{(VII.)}$$

Si deſideretur diſtantia Perihelii ſeu AS, quæ nominetur p, quum notum ſit eſſe $p\, (1 + e) = b$ vel $p = a\, (1 - e)$, ex formulis ſuperioribus eruitur:

$$p = \frac{R\, Cos\, x\, Sin\, \tfrac{1}{2} z}{Sin\, (\tfrac{1}{2} z + x)} \quad \text{(VIII.)}$$

Pari ratione, ſi diſtantia apſidis ſuperioris dicatur q, invenietur:

$$q = \frac{R\, Cos\, x\, Cos\, \tfrac{1}{2} z}{Cos\, (\tfrac{1}{2} z + x)} \quad \text{(IX.)}$$

Determinato igitur (§. V.) ſitu axeos ſeu ang. z, & computato ſecundum formulam III angulo x, invenietur ipſa Sectio Conica per binas quasvis harum formularum IV - - - IX.

Scholion I. Ad angulum illum x quod attinet, ex inſpectione formularum allatarum levi adhibita attentione colligitur, illum eſſe angulum, a radio SL & recta ad Sectionem Conicam in L normali comprehenſum.

Schol. 2. Si inventa ſectio Conica fuerit ellipſis, per notiſſimam legem KEPLERIANAM innoteſcet

 etiam

etiam tempus periodicum Planetæ in hac trajectoria incedentis. Posito enim tempore periodico planetæ $= P$ atque Telluris $= T$ (quod secundum Dn. De la Lande est $= 365^d\ 6^h\ 9'\ 11'' = 365^d, 256377$) assumtaque media Telluris a Sole distantia $= 1$, erit $P^2 : T^2 :: a^3 : 1$; adeoque $P = T\sqrt{a^3} = T\sqrt{\frac{R^3 \, Cos\, x^3 \, Sin\, (z + x)^3}{Sin\, (z + 2x)^3}}$.

§. VII.

Usum atque applicationem formularum, quas (§§. V. VI.) exhibuimus, quum nihil habere difficultatis autumemus, unicum iisdem illustrandis exemplum attulisse sufficiet. Cel. Dn. Prosperin (*Kongl. Sv. Vet. Acad. Handl. 1783 p. 181 - - 184*) invenerat novi Planetæ *Herscheliani* tres longitudines heliocentricas in orbita & his respondentes ejusdem distantias a Sole sequentes: A. 1781 m. Apr. $16^d\ 8^h$ Temp. med. ad Merid. Paris. Longitudinem $= 2^s\ 27^\circ\ 52'\ 0''$ & distantiam a Sole $= 18.9033$. A. 1782 m. Apr. $10^d\ 9^h\ 8'\ 38''$ Long. $= 3^s\ 2^\circ\ 11'\ 43''$, atque dist. $= 18.8710$. A. 1783 m. Apr. $13^d\ 20^h\ 57'\ 16''$ Long. $= 3^s\ 6^\circ\ 39'\ 23''$ & dist. $= 18.8382$: posita distantia terræ a Sole media $= 1$. Ut ex his datis determinetur orbita planetæ, sumantur differentiæ longitudinum (ob præcessionem æquinoctiorum debite correctarum) $4^\circ\ 18'\ 53'', 52$ & $8^\circ\ 45'\ 42'', 64$; quo facto calculus ita subducitur:

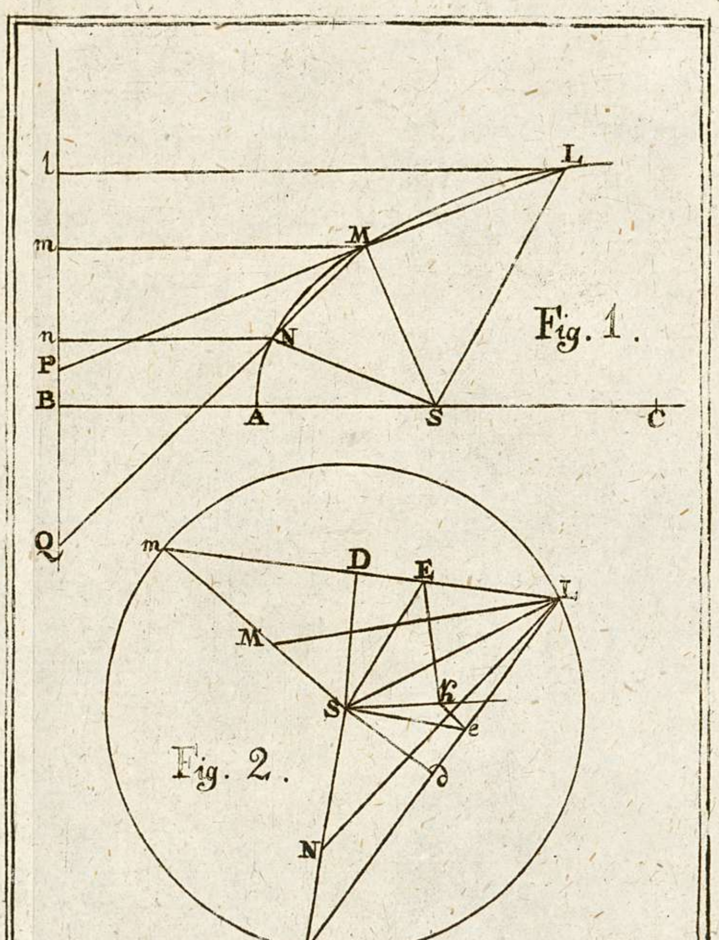
$R =$

$R = 18,9033$
$r = 18,8710$
$\varrho = 18,8382$
$R - r = 0,0323$
$R - \varrho = 0.0651.$
$u = 4^\circ\ 22'\ 51'',32$
$v = 2.\ 9.\ 26,76$
$\lambda = 45.\ 8.\ 36,20$
$45^\circ + \lambda = 90.\ 8\ 36,20$
$\frac{1}{2}u = 2.\ 11.\ 25,66$
$\frac{1}{2}v = 1.\ 4.\ 43,38$
$\frac{1}{2}(u-v) = 1.\ 6.\ 42,28$
$\frac{1}{2}(u+v) = 3.\ 16.\ 9,04$
$z + \frac{1}{2}(u+v) = 97.\ 20.\ 54,01$
$z = 94.\ 4.\ 44,97$
$z + v = 96.\ 14.\ 11,73.$
$x = 1.\ 18.\ 24,27$
$z + x = 95.\ 23.\ 9,24$
$z + 2x = 96.\ 41.\ 33,51$
$e = 0.0229001.$

$L(R-r) = \bar{2}.5092025$
$L\varrho = 1.2750394$
$L\,Sin\,u = \bar{2}.8830197$
$\bar{2}.6672616$

$L(R-\varrho) = \bar{2}.8135810$
$L\,r = 1.2757949$
$L\,Sin\,v = \bar{2}.5757121$
$\bar{2}.6650880$
$L\,Tg\,\lambda = 0.0021736$
$L\,Tg\,(45^\circ + \lambda) = 2.6016063$
$L\,Tg\,\frac{1}{2}\,u - v = \bar{2}.2879369$
$L\,Tg\,(z + \frac{1}{2}.u + v) = 0.8895432$
$L\,(R-r) = \bar{2}.5092025$
$L\,Sin\,z = \bar{1}.9988985$
$\bar{2}.5081010$

$L\,2 = 0.3010300$
$L\,r = 1.2757949$
$L\,Sin\,v = \bar{2}.5757121$
$L\,Sin\,(z+v) = \bar{1}.9974221$
0.1499591
$L\,Tg\,x = \bar{2}.3581419$
$L\,Sin\,x = \bar{2}.3580291$
$L\,Sin\,(z+x) = \bar{1}.9980782$
$L\,e = \bar{2}.3599509$



C.L.S. sc: